VII The disproof of undecidability of Gödel's proposition[1]

Dec 2022. Kurt Gödel constructed a proposition of the theory of natural numbers in 1931 for which neither a proof nor a refutation should exist, though it is true.
The true meaning of 0, "nothing"[2], discloses new prospects of proof. Propositions about "non-existence" can be proved by equivalent propositions about "nothing". Gödel's proposition, based on the "non-existence" of its proof, is proven and decided by "nothing" of proof. The theory of the natural numbers is complete, the axiomatic system that Gödel premised, is incomplete. It is completed by the axiom of "nothing" of proof.

1. Introduction

Kurt Gödel published a fundamental article "On formally undecidable propositions of Principia Mathematica and related systems I (1931)". He constructed a **proposition G of the theory of natural numbers, that says about itself that no axiomatically founded sequence of formulas exists that could prove it. For this proposition, Gödel demands in his first incompleteness theorem that neither a proof nor a refutation exists, although it is true.** The 2nd incompleteness theorem states that the consistency of the theory is also not provable. **Gödel further asserts that even additional axioms cannot lead to decidability of G.** This article however demonstrates that such an axiom can be formulated. The starting point is the equivalence of the terms "nothing" and "non-existence". Equivalent propositions about the existence of "nothing" and "non-existence" prove each other. Article I demonstrates that the 0 is not a number but represents "nothing" of the numbers. Reasoning, that 0 also represents "nothing" of proof of number theory, is supplied.
This results uncloses a new possibility of proof, that is essential for proof and decidability of Gödel's proposition. Crucially, "nothing", that was suppressed in mathematics in the 16th century because of reservations about the metaphysical Nothing, is reintroduced.

2. Gödel's proposition and the incompleteness theorems

A proof is built by a sequence of formulas (the sequent), the first of which is an axiom or rule, and the last is the proven theorem. **Gödel coded proof by natural numbers, the so-called Gödel numbers,** that are marked by ⌜ ⌝. A formula as well as a sequent are each mapped to a natural number. **G is defined by the assessment that all sequents cannot prove the proposition with the Gödel number ⌜G⌝.** So G makes a statement about itself, this is possible by citing the proposition through its Gödel number ⌜G⌝. Formally the definition is formulated by (1), expressed with the following meaning of the characters: $\forall$ (for all), $\neg$ (not), $\vdash$ (proof). A sequent has a Gödel number $x > 1$.

(1) $G \equiv \forall x > 1 : \neg\ (x > 1 \vdash ⌜G⌝)$

Supposing G is provable by a sequent contradicts the proposition itself. Supposing G is refutable then its negation would be provable, saying that there is a sequent that proves G. This statement is also contradictory.

The 1. incompleteness theorem states, that Gödel's proposition is undecidable by a sequent.

1 The topic including the bibliographical references are part of the book by Gert Treiber, *“Nichts”, Krise und reEvolution der Grundlagen der Mathematik,* Cuvillier Verlag, 2020.

2 Article I Empirical fact: 0 is not a number but represents “nothing“

Consistence is premised for the particular steps of the construction of G. Gödel's proposition results from consistency. A proof of consistency would therefore imply the proof of G. According to Gödel, this proof is not possible, so a proof of consistency is also impossible as stated by the 2. incompleteness theorem

3. Axiom of „nothing“ of proof

The proof of Gödel's proposition requires an axiom. The 0 is the point of origin of this requirement. The sign marks an empty place in the sequence of digits of place value numbers and in equations, representing "nothing" of numbers[2]. The place where a sequent is located is also empty if a proposition is not provable. The empty place of proof can be marked by 0 as well, representing “nothing” of proof. The definition of 0 can be extended to comprise this meaning too:
(2) Definition: 0 is the sign representing "nothing" of proof, "no sequent".
The axiom of the "nothing" of proof is formulated by (3). ∃ stands for “it exists”, $\mathcal{Z}$ is a sign, the Greek letter Γ indicates a sequent, φ_{np} denotes a proposition not provable by a sequent.
(3) $\exists\ \mathcal{Z}: \mathcal{Z} = 0 \vdash \neg (\exists \Gamma: \Gamma \vdash \varphi_{np})$[3]
The axiom is intuitively understandable. **The premise of the existence of "nothing" of the proof proves "non-existence" of a proof of φ_{np} by a sequent Γ.**
All false as well as Gödel's true proposition are propositions φ_{np}.

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4. Disproof of undecidability of Gödel's proposition

The axiom is applied to Gödel's proposition G and mapped to Gödel numbers.
The variable sign $\mathcal{Z}$ and the variable Γ are mapped to number variable x. According to Gödel, the 0 owns Gödel number 1[4]. (4) thus results from (3):
(4) $\exists x: x = 1\ \ \neg\ (\exists x > 1: x > 1 \vdash \ulcorner G \urcorner)$
The premise of existence of Gödel number x = 1 proves: It is not true that there exists a Gödel number x > 1 that would prove the proposition with Gödel number ⌜G⌝.
Gödel's proposition (1) can also be formulated equivalently by (5): There exists no sequent that could prove G:
(5) $G = \neg (\exists x > 1: x > 1 \vdash \ulcorner G \urcorner)$
This proposition agrees with the right-hand side of '⊢' in (4). **From (4) and (5) follows (6):**
(6) $\exists x: x = 1 \vdash G$
The existence of Gödel number 1 proves Gödel's proposition.
This surprising result deserves an explanation. To do this, (6) is decoded:
The existence of “nothing” of proof proves proposition G, formulated by the “non-existence” of its proof by a sequent.
Correspondingly, the proof of G is founded in “nothing”, i.e. “no sequent”.
The refutation of G would correspond to the proof of its negation.
Then $\exists x: x = 1 \vdash \exists x > 1: x > 1 \vdash \ulcorner G \urcorner$ should hold. This proposition is wrong.
The negation of G cannot be proven.
G is decided, the 1st incompleteness theorem is refuted.
The 2nd incompleteness theorem implies that the proof of consistency would result in the proof of G and that therefore a contradiction would be existent. The proof of G is no longer contradictory, so the proof of consistency is no longer impossible.
Gödel premised a sequent for his proof, i.e. x > 1, under this prerequisite the "incompleteness" theorems are retained. His proposition indeed cannot be decided from his axiomatic system. With his theorems, however, he did not prove the incompleteness of the theory of natural numbers. "Incompleteness" in connection with these theorems is therefore put in quotation marks. The

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3 In the terminology of the sequent calculus $\exists \mathcal{Z}: \mathcal{Z} = 0$ is the antecedent and $\neg (\exists \Gamma: \Gamma \vdash \varphi_{np})$ is the succedent

4 The 1 is the neutral element of reasoning coded by Gödel numbers. The factor 1 does not affect the Gödel number 'Γ' of the proof, the Gödel number 1 represents "no sequence", "no proof".

disproof incompleteness relies on x = 1, i.e. "no sequent". Gödel postulated that further axioms could not decide his theorem. The new axiom of ‘nothing’ of the proof refutes him.T**he theory of natural numbers is not incomplete, but the axiomatic system that Gödel premised.**

5. Comment

- According to the previous theory, Gödel's proposition is true but not provable. **Semantic truth** is therefore seen as the stronger concept compared to **syntactic provability.** This **difference can no longer be justified** by the "incompleteness" theorems.
- Current theory furthermore necessitates a non-standard model with "non-standard natural numbers”, n_{nst} , in which non-provability of G is "false" and hence G is "provable". The terms placed in quotation marks make it clear that there are questionable statements.They cannot be decoded, so the terms "false" and "provable" cannot be interpreted. The n_{nst} contain the standard natural numbers as a subset, but are essentially more extensive. According to the proof presented here, this obscure **non-standard model of natural numbers** does not exist. Since Gödel's proposition is both semantically true and syntactically provable, it **becomes needless.**
- Hilbert's programme was intended to prove that all theorems in mathematics were free of contradictions and decidable. The programme failed due to Gödel's incompleteness theorems. After the refutation of undecidability. **Hilbert's programme is possible again.**